Total Neutrino and Antineutrino Nuclear Cross Sections around 1 GeV

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based on a collaboration with O. Benhar

hep-ph/0610403

BENE meeting, November 14th

Plan of the talk

1 Introduction

present status of neutrino oscillations $\left\{ \begin{array}{c} \text{what we know} \;-\; \text{we do not know} \\ \text{future experimental facilities} \end{array} \right.$

2 Inclusive elastic neutrino-nucleus cross section

- the IA regime
- the (ν_e-^{16}O) cross sections
- comparison with other calculations

3 Resonance production

- importance of including resonances in the calculation
- total cross section

4 Conclusions and outlooks

Introduction

- **Present State of Neutrino Oscillation Parameters**

Solar Sector
$$\begin{cases} \Delta m^2_{12} \sim 8.2 \cdot 10^{-5}\ eV^2 \\ \sin^2 2\theta_{12} \sim 0.80 \end{cases}$$

Atm Sector
$$\begin{cases} \Delta m^2_{23} \sim 2.5 \cdot 10^{-3}\ eV^2 \\ \sin^2 2\theta_{23} > 0.9 \end{cases}$$

$\mathrm{sign}(\Delta m^2_{23})$?

$\mathrm{sign}(\sin 2\theta_{23})$?

STILL MISSING

θ_{13} poorly known ($\theta_{13} < 13^\circ$) $\longrightarrow$ 3 Family Oscillations ?

δ completely unknown $\longrightarrow$ Leptonic CP-violation ?

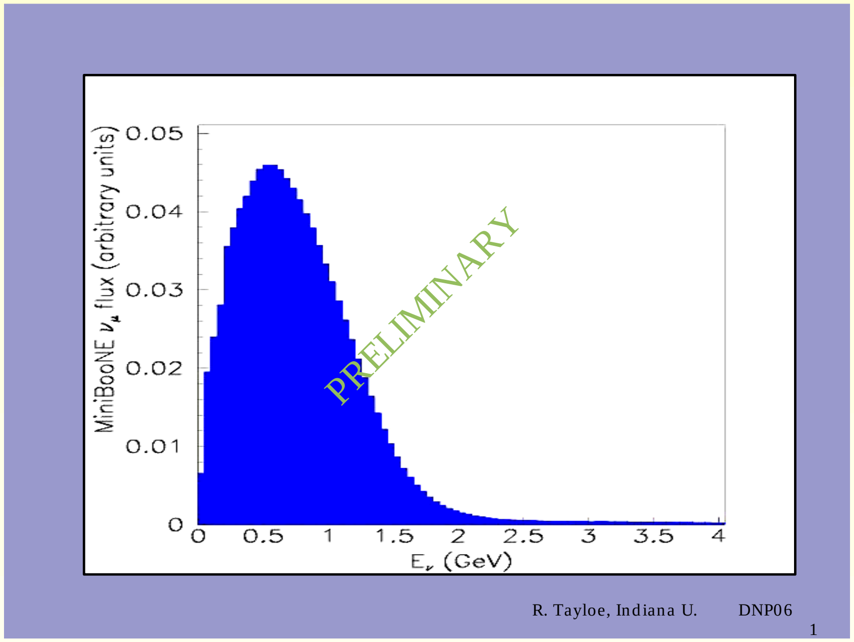
High precision neutrino experiments required

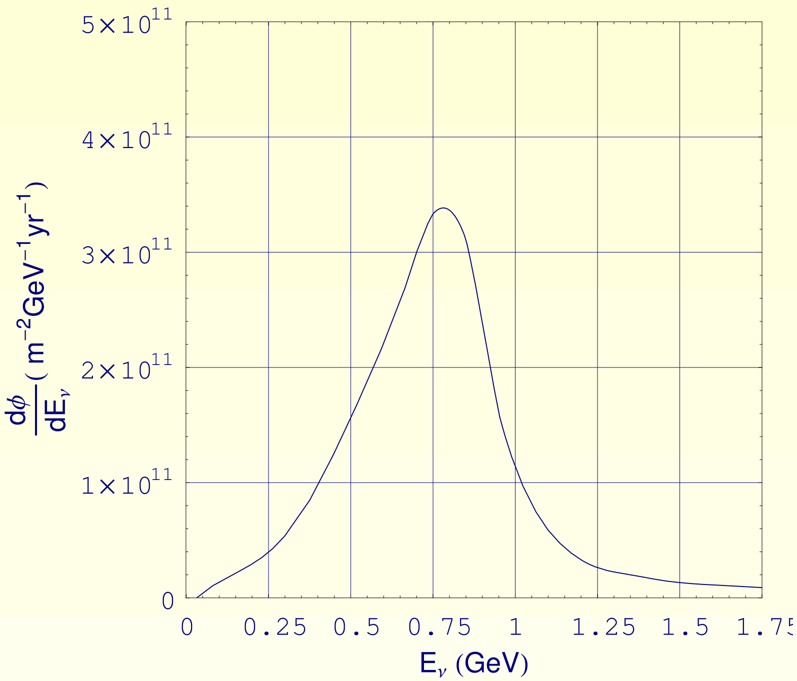
to fully understand the neutrino mixing parameters

Why cross sections around 1 GeV

many current and planned experiments use a ν flux picked at $\sim$ 1 GeV

MiniBoone **T2K-I**





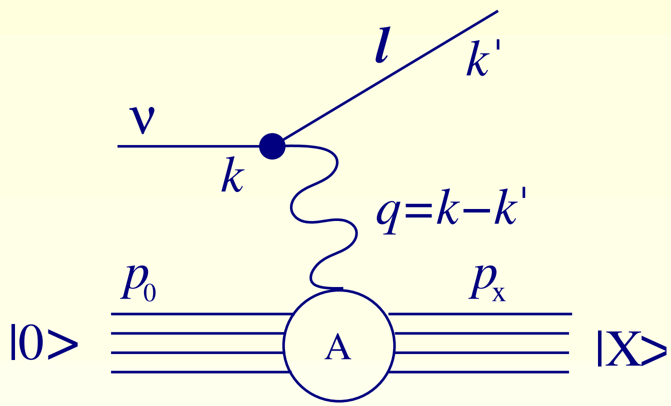
and many others (NOνA, high γ β-beams...)

- **very few neutrino scattering data**
- **data have generically not been taken on the same targets used in the experiments**

important to know very precisely the ν-<u>nucleus</u> cross sections at $E_\nu \sim$ 1 GeV

The cross sections

- at low energies ($E_\nu \leq 0.6 - 0.7$ GeV): the dominant contribution comes from **quasi-elastic** scattering;
- at higher energies: **inelastic production** of charged leptons (via resonance excitation) + inelastic production of π^0 also contribute
- negligible deep inelastic scattering contribution
- formalism to describe inclusive $\nu + A \to l + X$ reaction



$$\frac{d^2\sigma}{d\Omega dE_l} = \frac{1}{16\,\pi^2}\,\frac{|\mathbf{k}^{'}|}{|\mathbf{k}|}\,L_{\mu\nu}W_A^{\mu\nu}$$

$$L^{\mu\nu} = 8\left[k^{'}_{\mu}\,k_{\nu} + k^{'}_{\nu}\,k_{\mu} - (k\cdot k^{'}) - i\,\varepsilon_{\mu\nu\alpha\beta}\,k^{'\,\beta}\,k^{\alpha}\right]$$

$$W_A^{\mu\nu} = \sum_X \langle 0|J_A^{\mu\,\dagger}|X\rangle\langle X|J_A^{\nu}|0\rangle\delta^{(4)}(p_0 + q - p_X)$$

The cross sections

the problem is the calculation of the hadronic tensor $W_A^{\mu\nu}$

- for $|\mathbf{q}| < 0.5\,\mathrm{GeV}$ NMBT + nonrelativistic wave functions + expansion of the current operator in powers of $|\mathbf{q}|/m_N$
  Carlson&Schiavilla, Rev. Mod. Phys. 70, 743 (1998)

- for larger $|\mathbf{q}|$ (to which we are interested in) we can no longer describe the final states $|X\rangle$ in terms of nonrelativistic nucleons

$$\Downarrow$$

we need a set of simplifying assumptions to describe relativistic motion of final state particles and the occurrence of inelastic processes

$\boxed{\textbf{the Impulse Approximation}}$

target nucleus seen as a collection of individual nucleons
$J_\mu \to \sum_i j_\mu^i$

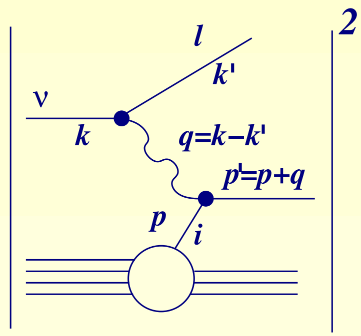
scattered nucleons and recoiling system $\mathcal{R}$ evolve independently of one another
$|X\rangle \to |i, p^{'}\rangle \otimes |\mathcal{R}, p_{\mathcal{R}}\rangle$
(no Final State Interactions)

The cross sections

Benhar et al.,

Phys.Rev.D72:053005,2005

$\sigma \sim \sum_i$



$$W_A^{\mu\nu} \quad = \quad \frac{G_F^2\, V_{ud}^2}{2} \int d^3p\, dE\, P(\mathbf{p}, E) \frac{1}{4\, E_{\mathbf{p}}\, E_{|\mathbf{p}+\mathbf{q}|}}\, W^{\mu\nu}(\tilde{p}, \tilde{q})\, \delta(\nu - E - E_{|\mathbf{p}+\mathbf{q}|})$$

- $P(\mathbf{p}, E)$ is the spectral function: probability distribution of finding a nucleon with momentum $\mathbf{p}$ and removal energy E in the target nucleus

it encodes all the informations about the initial struck particle

- $W^{\mu\nu}$ describes electroweak interactions of the i-th nucleon in free space
- effect of nuclear binding of the struck nucleon is accounted for by the replacement
  $\boxed{q = (\nu, \mathbf{q}) \rightarrow \tilde{q} = (\tilde{\nu}, \mathbf{q})}$ with $\tilde{\nu} = E_{|\mathbf{p}+\mathbf{q}|} - E_{\mathbf{p}}$

  (a fraction $\nu - \tilde{\nu}$ goes into excitation energy of the spectator system)

$\Rightarrow$

The cross sections

then we get

$$\boxed{\frac{d^2\sigma_{IA}}{d\Omega dE_l} = \int d^3p\, dE\, P(\mathbf{p},E)\, \frac{d^2\sigma_{\rm elem}}{d\Omega dE_l}\, \delta(\nu - E - E_{|\mathbf{p}+\mathbf{q}|})}$$

$$\frac{d^2\sigma_{\rm elem}}{d\Omega dE_l} = \frac{G_F^2\, V_{ud}^2}{32\,\pi^2}\, \frac{|k^{'}|}{|k|}\, \frac{1}{4\, E_{\mathbf{p}}\, E_{|\mathbf{p}+\mathbf{q}|}}\, L_{\mu\nu} W^{\mu\nu}$$

- The hadronic tensor is decomposed in structure functions as usual

$$\begin{aligned} W^{\mu\nu} \quad &= \quad -g^{\mu\nu}\, W_1 + \tilde{p}^{\mu}\, \tilde{p}^{\nu}\, \frac{W_2}{m_N^2} + i\, \varepsilon_{\mu\nu\alpha\beta}\, \tilde{q}^{\alpha}\, \tilde{p}^{\beta}\, \frac{W_3}{m_N^2} + \tilde{q}^{\mu}\, \tilde{q}^{\nu}\, \frac{W_4}{m_N^2} + \\ & \quad (\tilde{p}^{\mu}\, \tilde{q}^{\nu} + \tilde{p}^{\nu}\, \tilde{q}^{\mu})\, \frac{W_5}{m_N^2} \end{aligned}$$

- the formalism can be applied to **both** elastic and anelastic processes specifying the form of the structure functions W_i

Elastic interactions

1- tensor contraction

$$L^{\mu\nu}\,W_{\mu\nu} \quad = \quad 16\sum_i W_i\left(\frac{A_i}{m_N^2}\right)$$

$$A_1 \quad = \quad m_N^2\,(k\cdot k^{'})$$

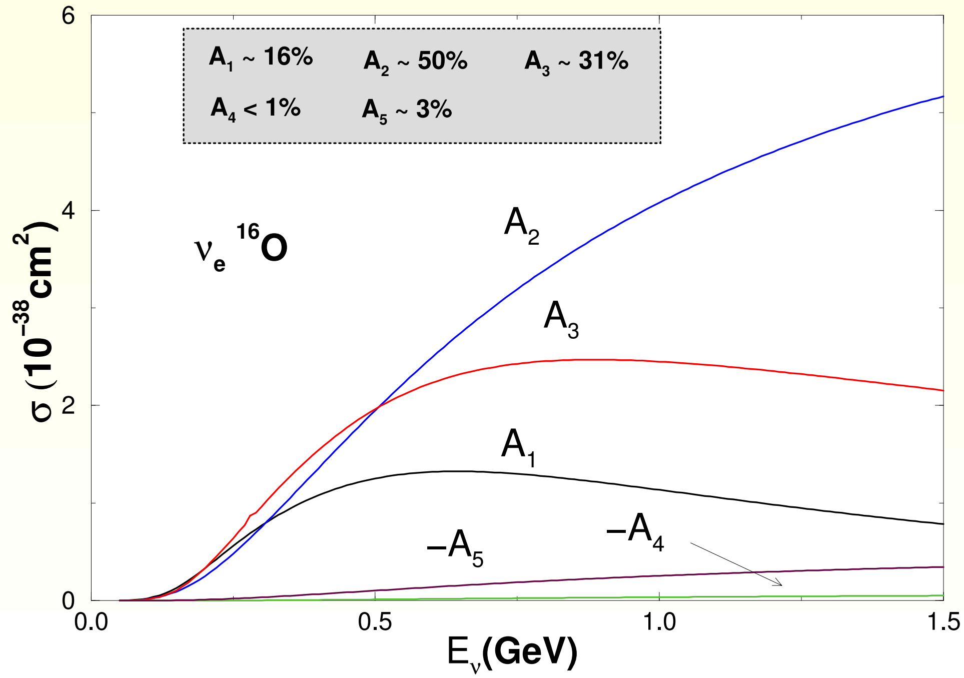
$$A_2 \quad = \quad (k\cdot\tilde{p})\,(k^{'}\cdot\tilde{p}) - \frac{m_N^2}{2}\,A_1$$

$$A_3 \quad = \quad (k\cdot\tilde{p})\,(k^{'}\cdot\tilde{q}) - (k\cdot\tilde{q})\,(k^{'}\cdot\tilde{p})$$

$$A_4 \quad = \quad (k\cdot\tilde{q})\,(k^{'}\cdot\tilde{q}) - \frac{\tilde{q}^2}{2}\,A_1$$

$$A_5 \quad = \quad (k\cdot\tilde{p})\,(k^{'}\cdot\tilde{q}) + (k^{'}\cdot\tilde{p})\,(k\cdot\tilde{q}) - (\tilde{q}\cdot\tilde{p})\,A_1$$

- A_i *reduce to the usual coefficients for free nucleons in the limit* $(\tilde{p},\tilde{q})\to(p,q)$
- $A_{4,5}$ *DO NOT DEPEND linearly on* m_l
- $W_i = W_i(F_1, F_2, F_A, F_P)$
- *dipole structure for form factors*



Elastic interactions

2- the spectral function $P(\mathbf{p}, E)$

- we analyzed different models for $P(\mathbf{p}, E)$ for ${}^{16}O(\nu_e, e^-)$

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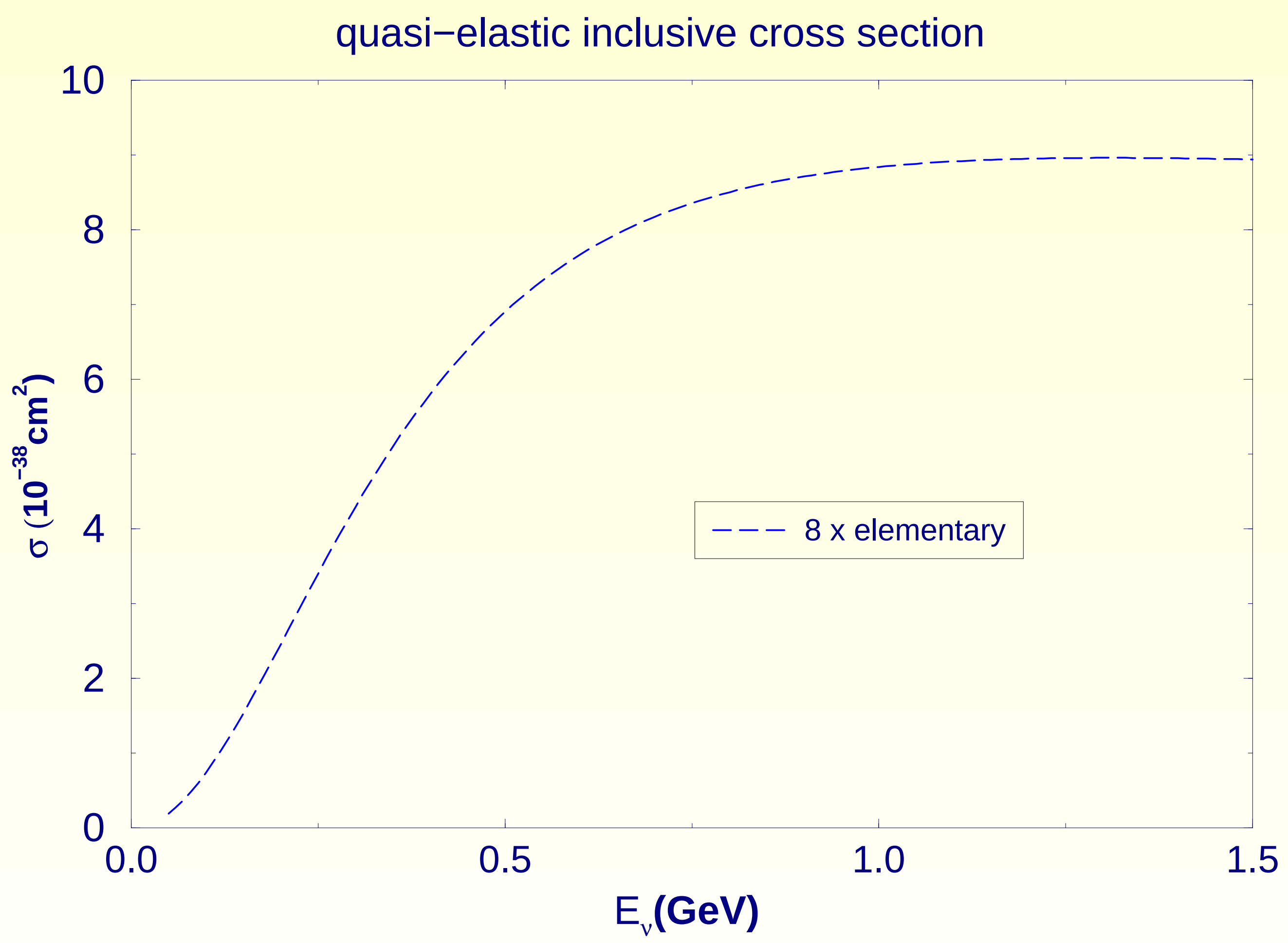
to start: the elementary cross section



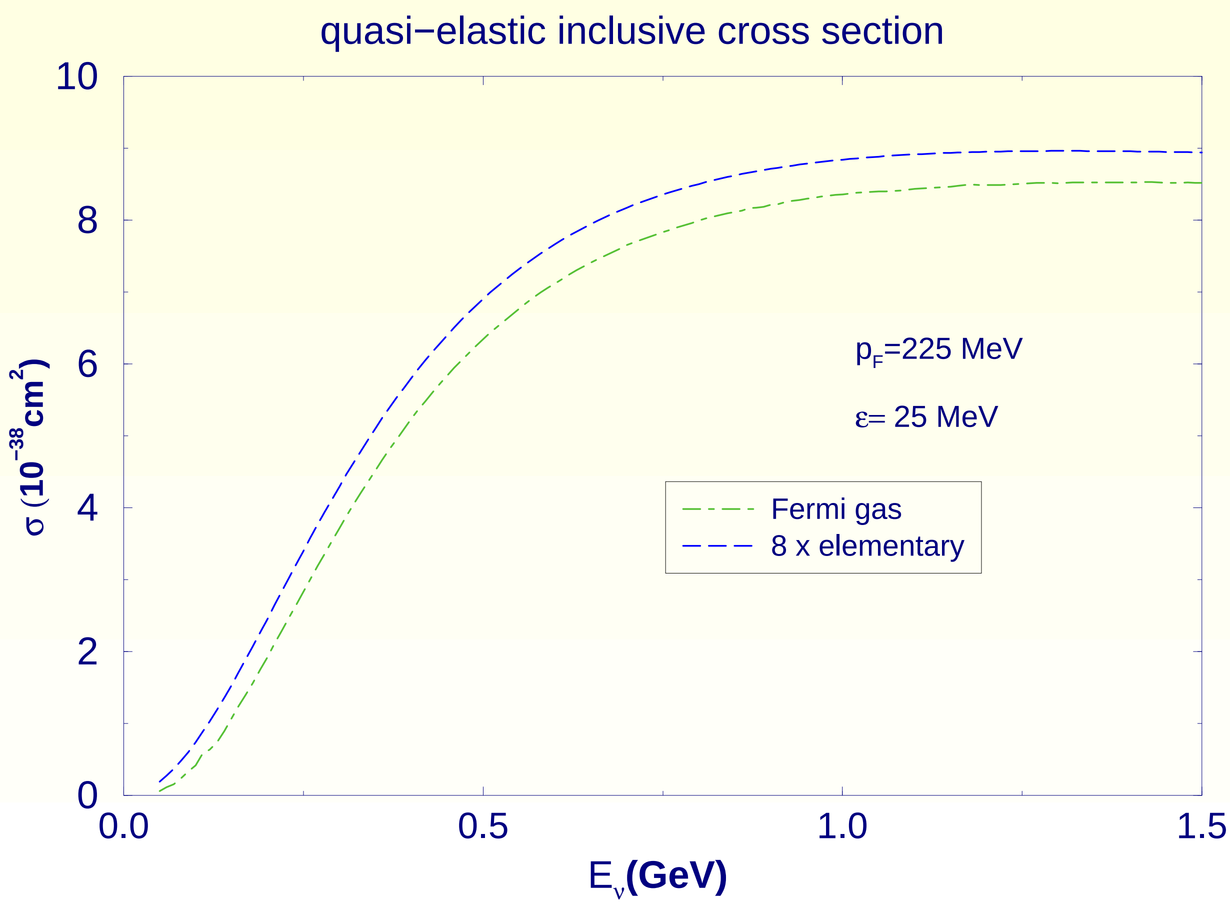
Elastic interactions

2- the spectral function $P(\mathbf{p}, E)$

- we analyzed different models for $P(\mathbf{p}, E)$ for $^{16}O(\nu_e, e^-)$

a simple prescription: the Fermi gas
$$\boxed{P(\mathbf{p}, E) = \left(\frac{6\,\pi^2\,A}{p_F^3} \right) \theta(p_F - \mathbf{p})\, \delta(E_{\mathbf{p}} - \varepsilon + E)}$$

where p_F is the Fermi momentum and ε is the average binding energy



- reduction of x-sect due to cut in nucleon momentum

Elastic interactions

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realistic spectral function

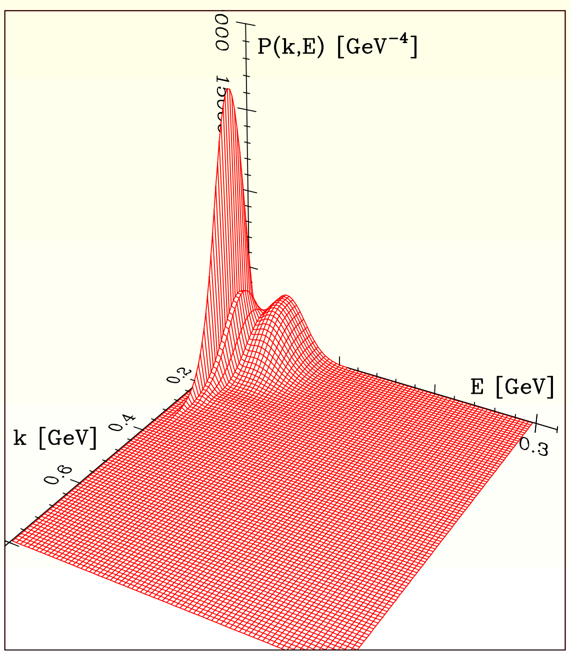
Benhar et al.,

Nucl. Phys. A 579 (1994) 493

Phys. Rev D72 (2005) 053005

- overwhelming evidence from electron scattering that the energy-momentum distribution of nucleons in the nucleus is quite different from that predicted by Fermi gas
- the most important feature is the presence of strong nucleon-nucleon (NN) correlations (virtual scattering processes leading to the excitation of the participating nucleons to states of energy larger than the Fermi energy)

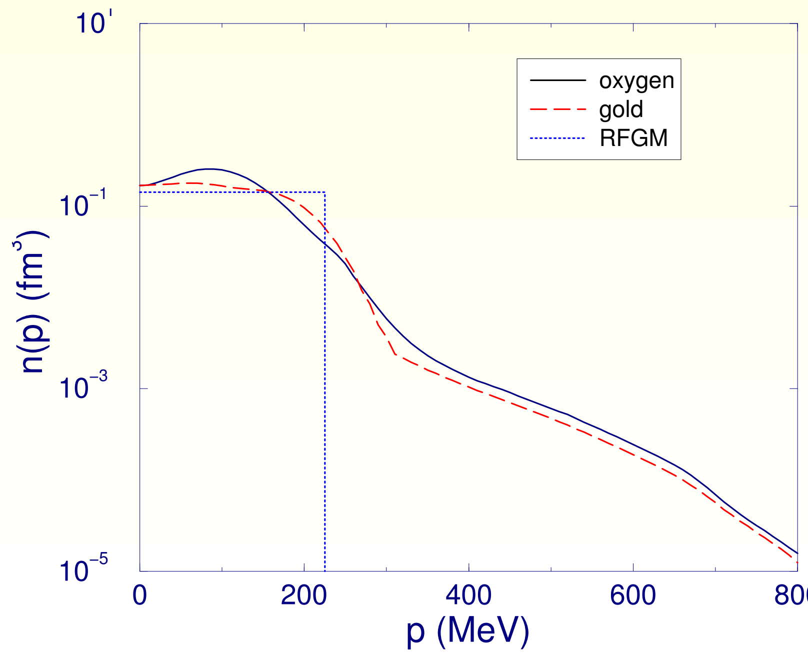
spectral function extends to $|\mathbf{p}| \gg p_F$ *and* $E \gg \varepsilon$



momentum distribution

$$n(\mathbf{p}) = \int dE \; P(\mathbf{p}, E)$$

$\Longrightarrow$



Elastic interactions

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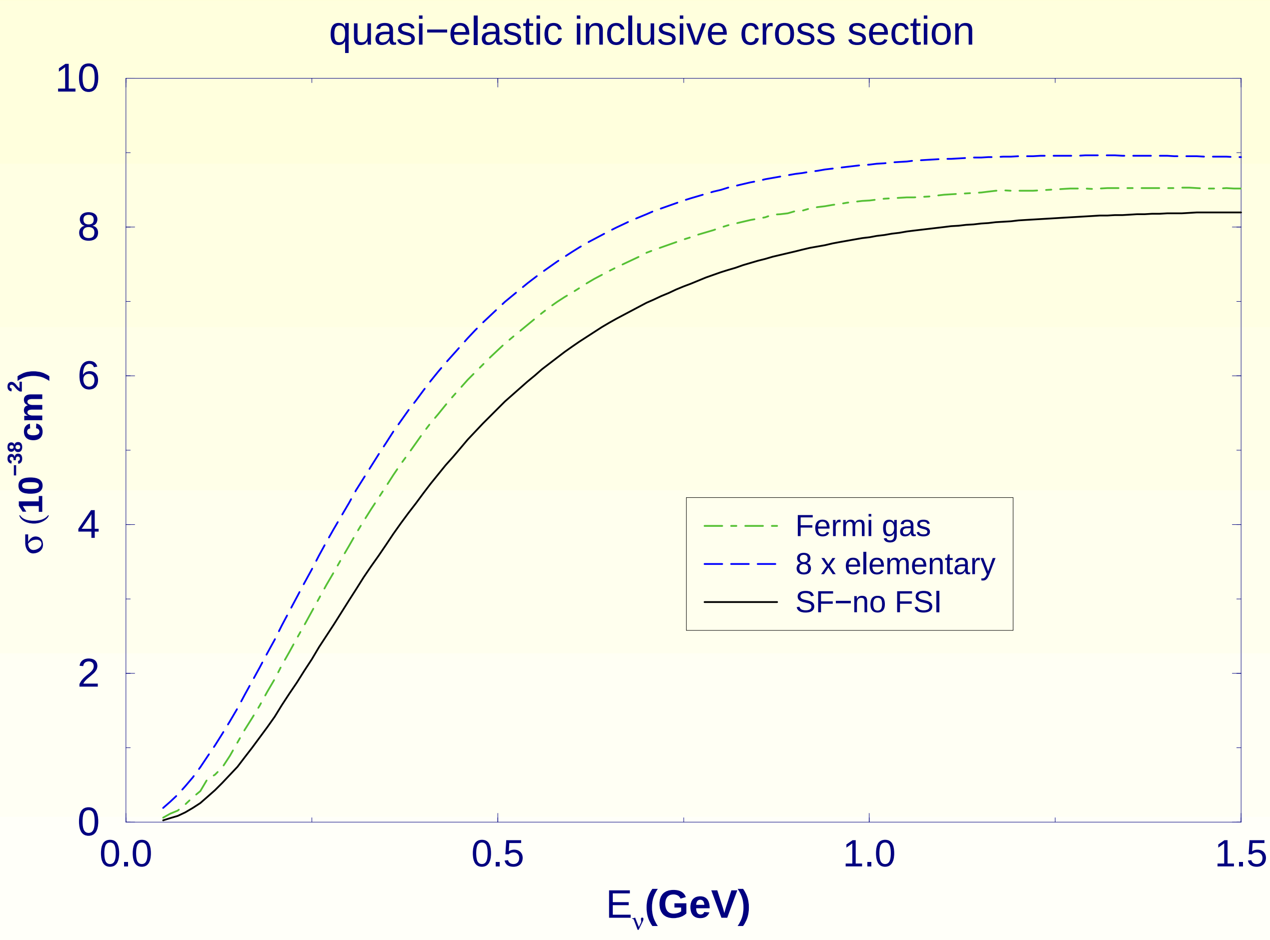
realistic spectral function **Benhar et al., Nucl. Phys. A 579 (1994) 493**



- further reduction from binding energy and short-distance NN correlations

Elastic interactions

- realistic description must take into account interaction of struck nucleon with the spectator system

$$\Downarrow$$

introduction of Final State Interaction (FSI)

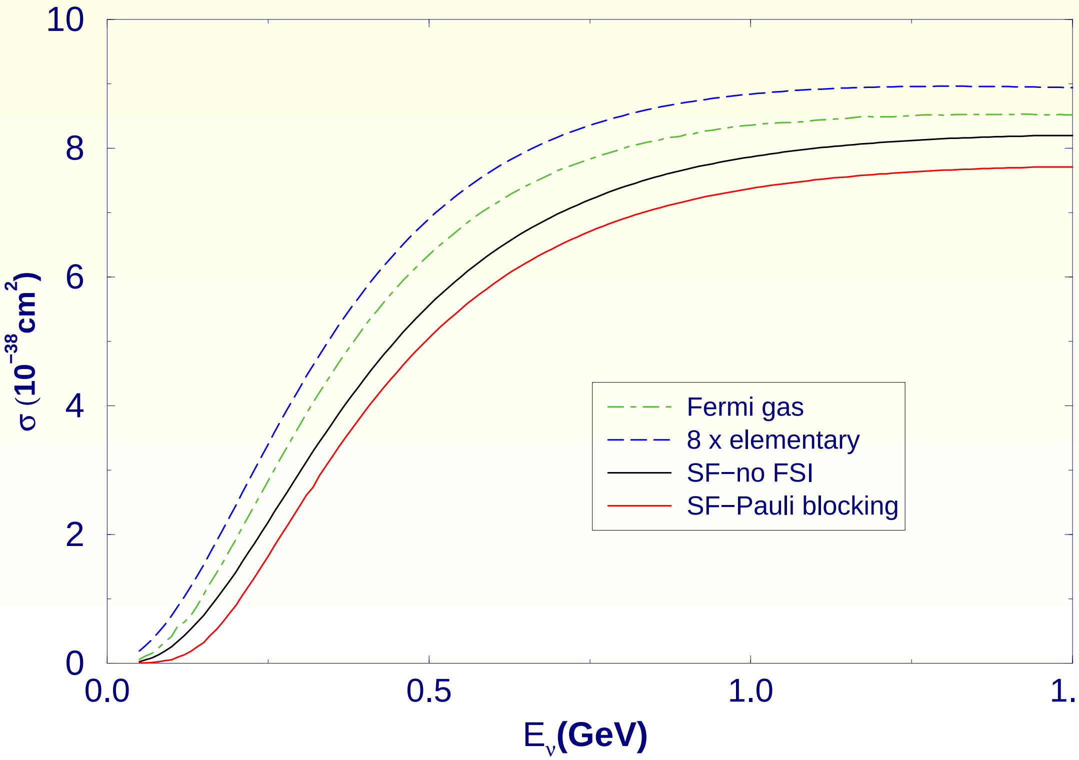
- a simple prescription to include statistical correlation: Pauli blocking

$$P(\mathbf{p}, E) \rightarrow P(\mathbf{p}, E)\, \theta(|\mathbf{p}+\mathbf{q}| - \bar{p}_F)$$

$$\bar{p}_F \quad = \quad \text{average nuclear Fermi momentum} = \int d^3r\, \rho_A(\mathbf{r})\, p_F(\mathbf{r}) = 209\, MeV$$

$$p_F(\mathbf{r}) \quad = \quad \left[3\,\pi^2\,\rho_A(\mathbf{r})/2\right]^{1/3} \qquad \rho_A(\mathbf{r}) = \text{nuclear density}$$

quasi−elastic inclusive cross section

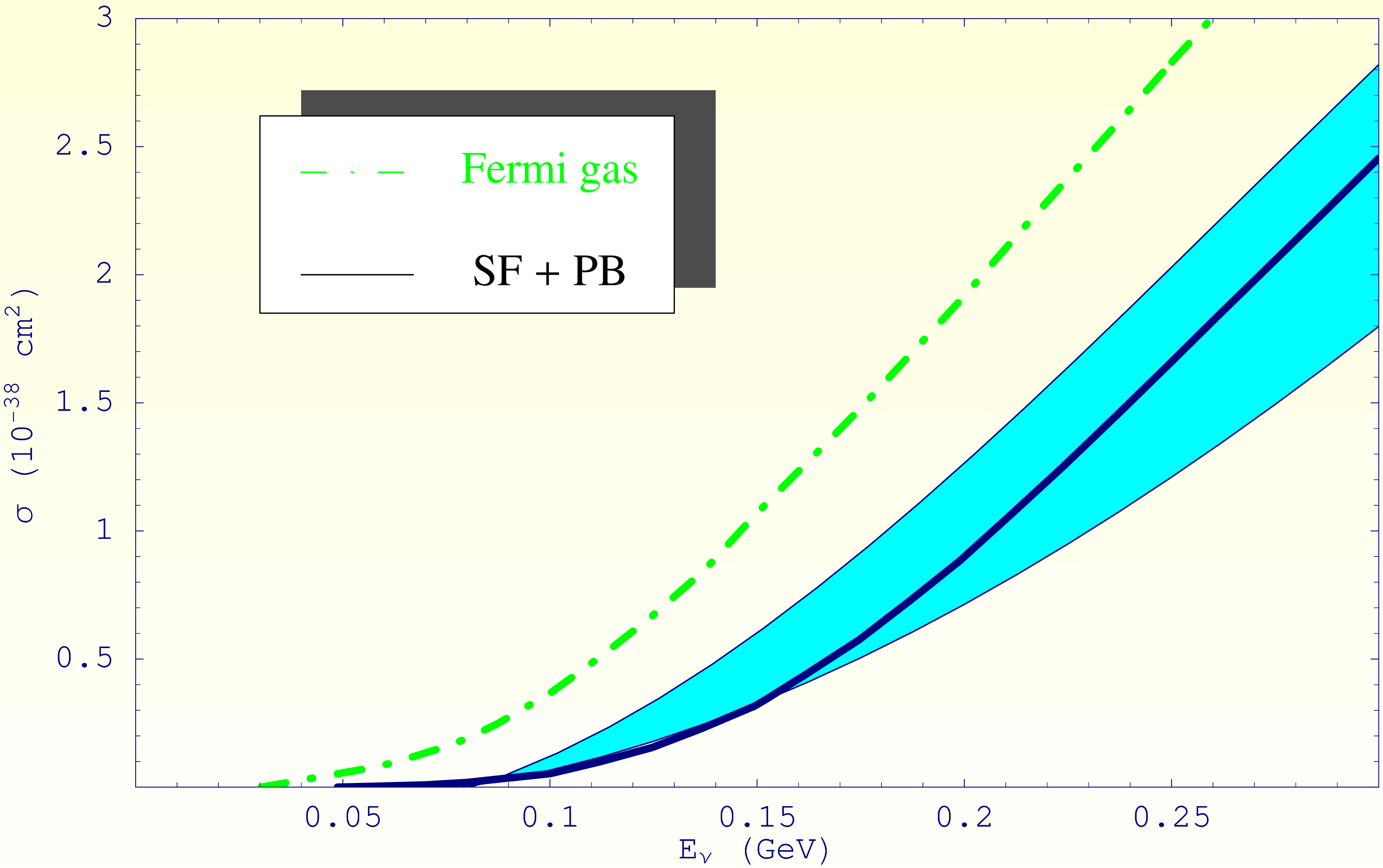


- Pauli blocking reduces the available phase space for knocked nucleon

differences between Fermi gas (with Pauli blocking) and our (red) prediction ranges from 15% to 4%

Elastic interactions
Comparison with other calculations

- most of them in the low (and very low) neutrino energy region
  J.E. Amaro et al, Phys.Rev.C70:055503,2004, Erratum-ibid.C72:019902,2005: collection of cross-sections on ^{16}O including FSI, RPA (using an effective Nucleon-Nucleon force) and Coulomb distortion

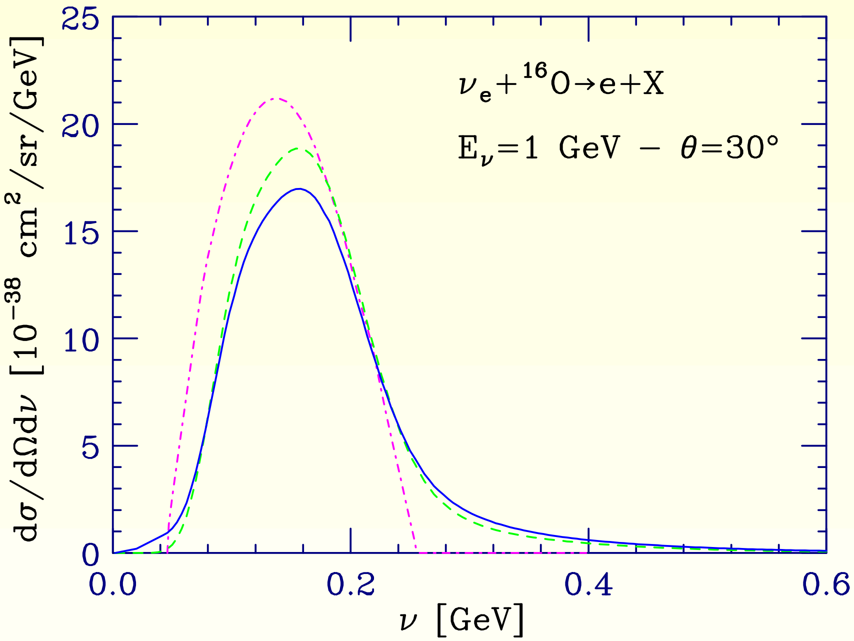


good agreement with cross sections including SF + Pauli blocking
the Fermi gas model is unsatisfactory

About Final State Interactions

In quasi-elastic inclusive processes *dynamical* FSI are weak:

- an energy shift of the differential cross section, due to the fact that the struck nucleon feels the mean field generated by the spectator particles
- a redistribution of the strength, leading to the quenching of the peak and the enhancement of the tails



- dashed line: Impulse Approximation
- solid line: Impulse Approximation + FSI
- dotted line: Fermi Gas

Benhar et al.,Phys. Rev D72 (2005) 053005

$\Downarrow$

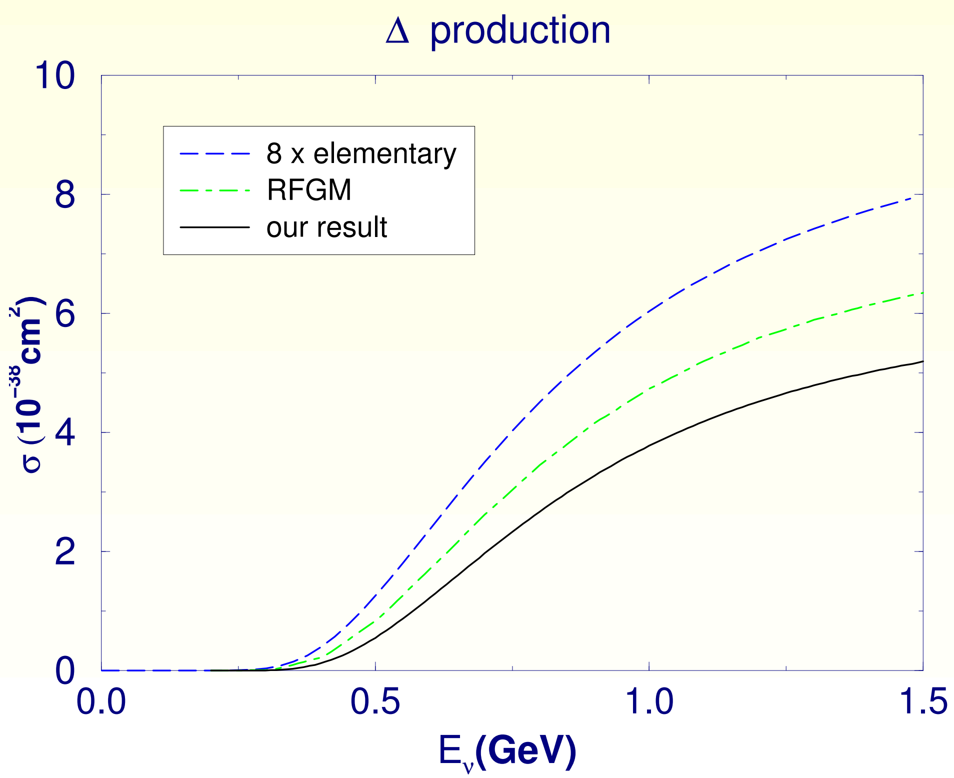
FSI do not affect the total inclusive cross section, resulting from integration over the lepton variables

Resonance production

- we are interested in the reactions $\nu\, n \to l^-\, \Delta^+$ and $\nu\, p \to l^-\, \Delta^{++}$

we use the same formalism as above where:

- the coefficients A_i are the same
- structure functions and form factors (magnetic dominance approximations) for $\langle \Delta^{++} | J_\mu | p \rangle$ are taken from
  Lalakulich et al., Phys Rev. D 71, 074003 (2005)
- we used isospin relation to obtain $\langle \Delta^{++} | J_\mu | p \rangle = \sqrt{3}\, \langle \Delta^+ | J_\mu | n \rangle$



- strong reduction with respect to both the elementary and RFGM calculations
- at $E_\nu = 1$ GeV

$$\boxed{\sigma_{SF}/\sigma_{RFGM} \sim 0.8}$$

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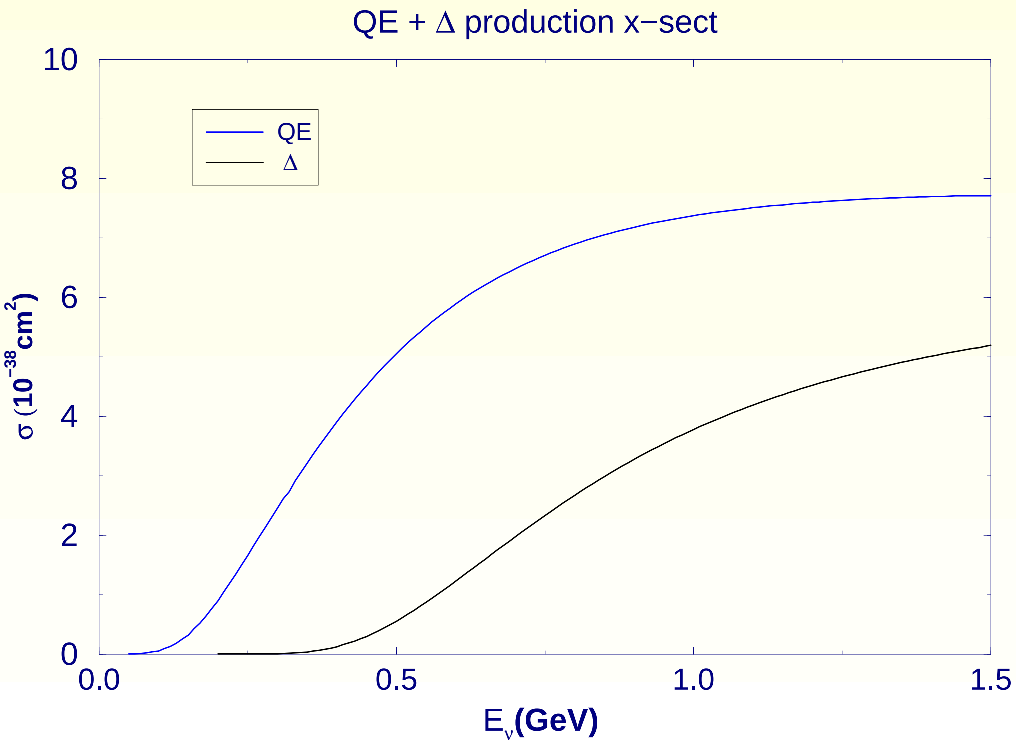
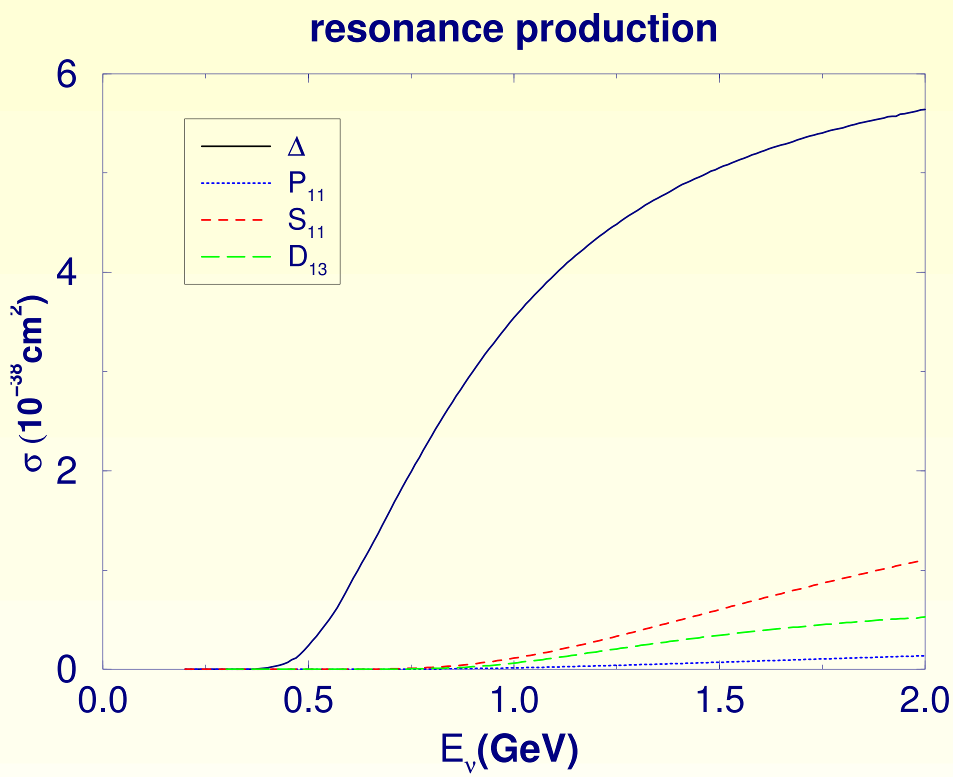


- contribution from Δ production turns out to be important for $E_\nu > 0.5$ GeV

Resonance production

- we also evaluate the impact on the cross section of including higher resonances
  Lalakulich et al., Phys.Rev. D74,014009 (2005)

- three isospin 1/2 states: $P_{11}(1440), D_{13}(1520), S_{11}(1535)$

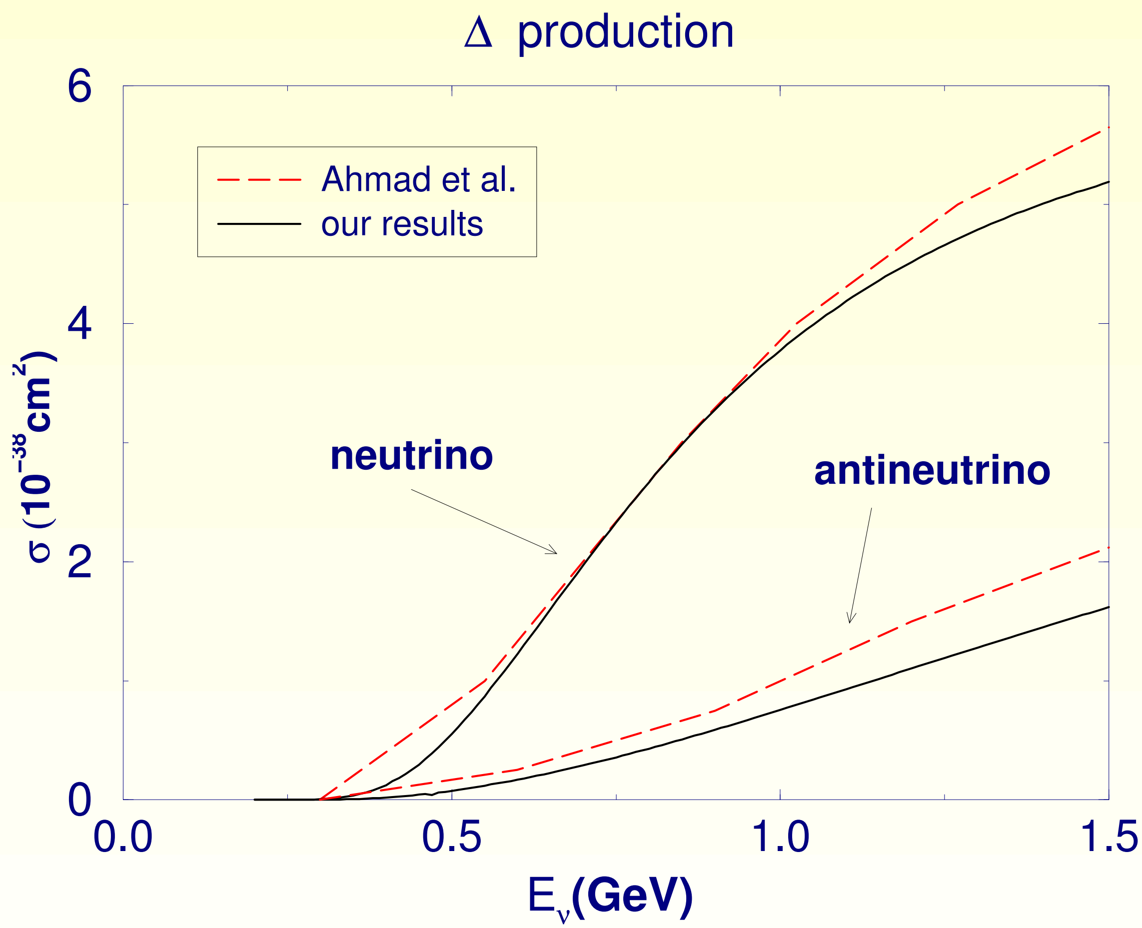


- as expected at the energies under discussion the second resonance region can be neglected

- $\sigma_\Delta : \sigma_{S_{11}} : \sigma_{D_{13}} : \sigma_{P_{11}} = 1 : 0.12 : 0.06 : 0.02$ at $E = 1.5$ GeV

Resonance production
Comparison with other calculations

- many calculations are devoted to exlusive pion production
- we compare our results with those of
  S. Ahmad et al., **nucl-th/0603001**



- the results seem to be in agreement

Comparison with experimental data

preliminary results

- from our results and standard isospin analysis we can estimate the cross section for $1\,\pi^+$

$$
\begin{aligned}
\mathcal{A}(\nu_\ell + p \to \ell^- + p + \pi^+) &= \mathcal{A}_3 \\
\mathcal{A}(\nu_\ell + n \to \ell^- + n + \pi^+) &= \frac{1}{3}\mathcal{A}_3 + \frac{2\sqrt{2}}{3}\mathcal{A}_1 \\
\mathcal{A}(\nu_\ell + n \to \ell^- + p + \pi^0) &= -\frac{\sqrt{2}}{3}\mathcal{A}_3 + \frac{2}{3}\mathcal{A}_1
\end{aligned}
$$

$\mathcal{A}_3$ is the amplitude for the isospin $3/2$ state of the πN system (predominantly the Δ)
$\mathcal{A}_1$ is the amplitude for the isospin $1/2$ state.

$$\Downarrow$$

$$\sigma_{\pi^+} = \frac{10}{9}\,\sigma_{\Delta^{++}} + \frac{8}{9}\,(b_1\,\sigma_{P_{11}} + b_2\,\sigma_{D_{13}} + b_3\,\sigma_{S_{11}})$$

b_i are branching ratios for π^+

Comparison with experimental data

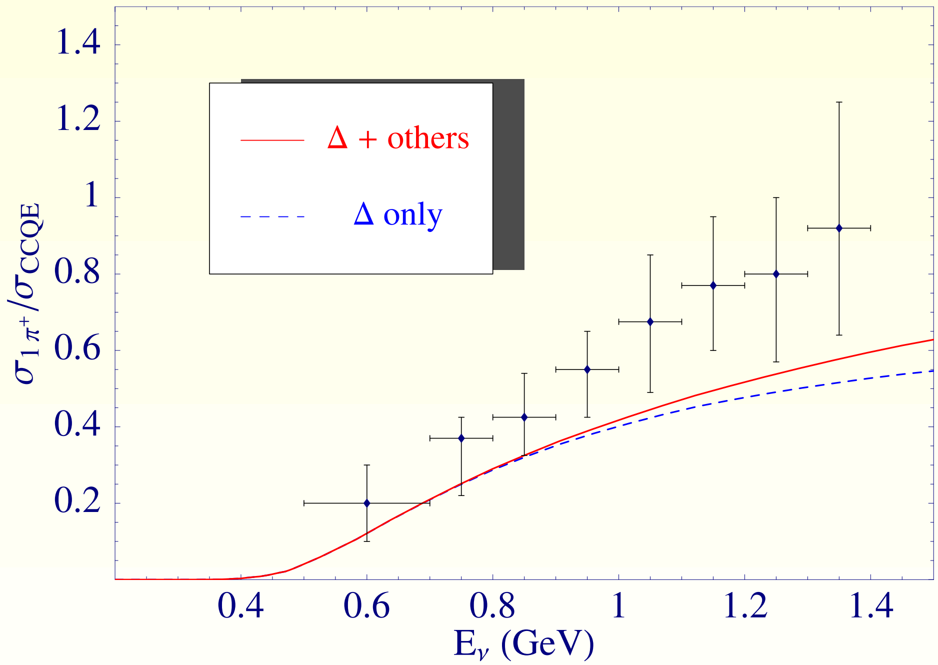
preliminary results

- from our results and standard isospin analysis we can estimate the cross section for $1\ \pi^+$
- the MiniBooNE collaboration has presented a preliminary measurement of

$$R = \frac{\sigma_{\pi^+}}{\sigma_{CCQE}}$$

in $(\nu_\mu, {}^{12}C)$ production

Wasko et al., Nucl.Phys.Proc.Suppl.159:50-55,2006



- theoretical curves underestimate the (preliminary) data. Effects due to non-resonant background and coherent pion production could be sizeable

Conclusions and outlooks

- the ν-nucleus cross-sections in IA regime with realistic spectral function have been discussed
- at $E_\nu \sim \mathcal{O}$(1 GeV) the quasi-elastic x-section with spectral function is lower than the widely applied Fermi Gas models; the inclusion of Pauli blocking further reduces the cross section
- the contribution of Δ-production is not negligible
- other resonances have been added, resulting in a $\sim$10% contribution to the resonance production cross section
- our results seem to be in agreement with other calculations
- future prospects:
  - inclusion of exclusive channels, both in the QE and resonance production regions
  - proper traitment of FSI (for differential CC x-sect and π-rescattering)
  - inclusion of non-resonant pion production

backup slides

Elastic Structure Functions and Form Factors

- structure functions

$$
\begin{aligned}
W_1 &= 2\left[-\frac{q^2}{2}\,(F_1+F_2)^2+\left(2\,m_N^2-\frac{q^2}{2}\right)\,F_A^2\right]\\
W_2 &= 4\left[F_1^2-\left(\frac{q^2}{4\,m_N^2}\right)\,F_2^2+F_A^2\right]\\
W_3 &= -4\,(F_1+F_2)\,F_A\\
W_4 &= -2\left[F_1\,F_2+\left(2\,m_N^2+\frac{q^2}{2}\right)\,\frac{F_2^2}{4\,m_N^2}+\frac{q^2}{2}\,F_P^2-2\,m_N\,F_P\,F_A\right]\\
W_5 &= \frac{W_2}{2}
\end{aligned}
$$

- form factors

$$
\tau = \frac{q^2}{4\,m_N^2} \qquad G_E = \frac{1}{\left(1-\frac{q^2}{M_V^2}\right)^2} \qquad G_M = 4.71\,G_E
$$

$$
F_1 = \frac{1}{1-\tau}\,(G_E-\tau\,G_M) \qquad F_2 = \frac{1}{1-\tau}\,(-G_E+G_M)
$$

$$
F_A = -\frac{1.26}{\left(1-\frac{q^2}{M_A^2}\right)^2} \qquad F_p = -\frac{1.28}{\left(1-\frac{q^2}{0.14}\right)^2}\,\frac{F_A}{-1.27}
$$

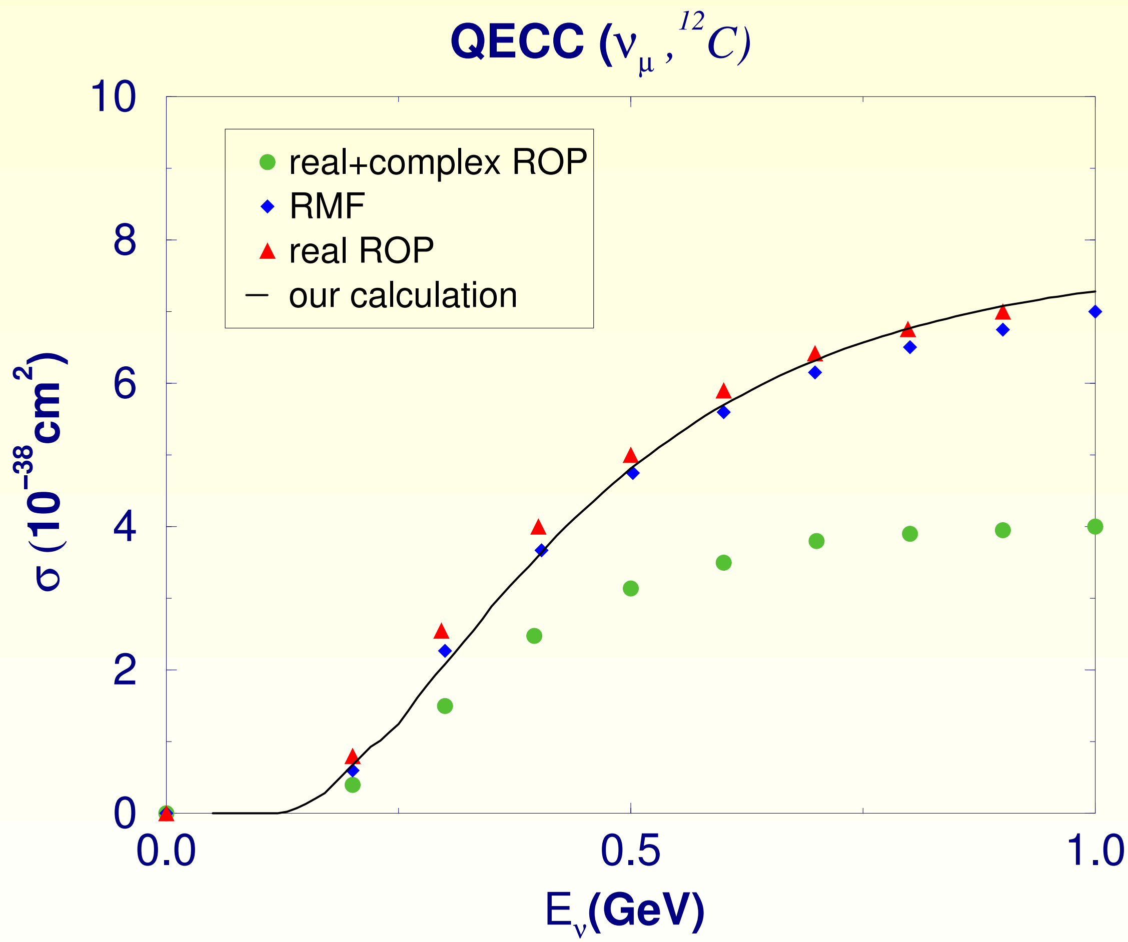
Anelastic Structure Functions and Form Factors

- structure functions are complicated functions of form factors
- detailed formulae in:
  Lalakulich et al., Phys.Rev. D**71**, 074003 (2005) and
  Lalakulich et al., Phys.Rev. D**74**, 014009 (2006)
- form factors for Δ-production

$$
\begin{aligned}
f_\pi &= 0.97\, m_\pi \qquad g_D = 15.3 \qquad M_V = 0.84 \qquad M_A = 1.05 \\
C_3^V(0) &= 1.95 \qquad C_3^V(q^2) = \frac{C_3^V(0)}{\left(1 - \frac{q^2}{M_V^2}\right)^2} \frac{1}{\left(1 - \frac{q^2}{4\, M_V^2}\right)} \\
C_4^V(q^2) &= -C_3^V(q^2)\,\frac{m_N}{W} \qquad C_5^V = 0 \\
C_5^A(0) &= \frac{f_\pi\, g_D}{\sqrt{3}} \qquad C_3^A = 0 \qquad C_5^A(q^2) = \frac{C_5^A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2} \frac{1}{\left(1 - \frac{q^2}{3\, M_A^2}\right)} \\
C_4^A(q^2) &= -\frac{C_5^A}{4} \qquad C_6^A(q^2) = C_5^A\,\frac{m_N^2}{-q2 + m_\pi^2}
\end{aligned}
$$

Elastic interactions
Comparison with other calculations

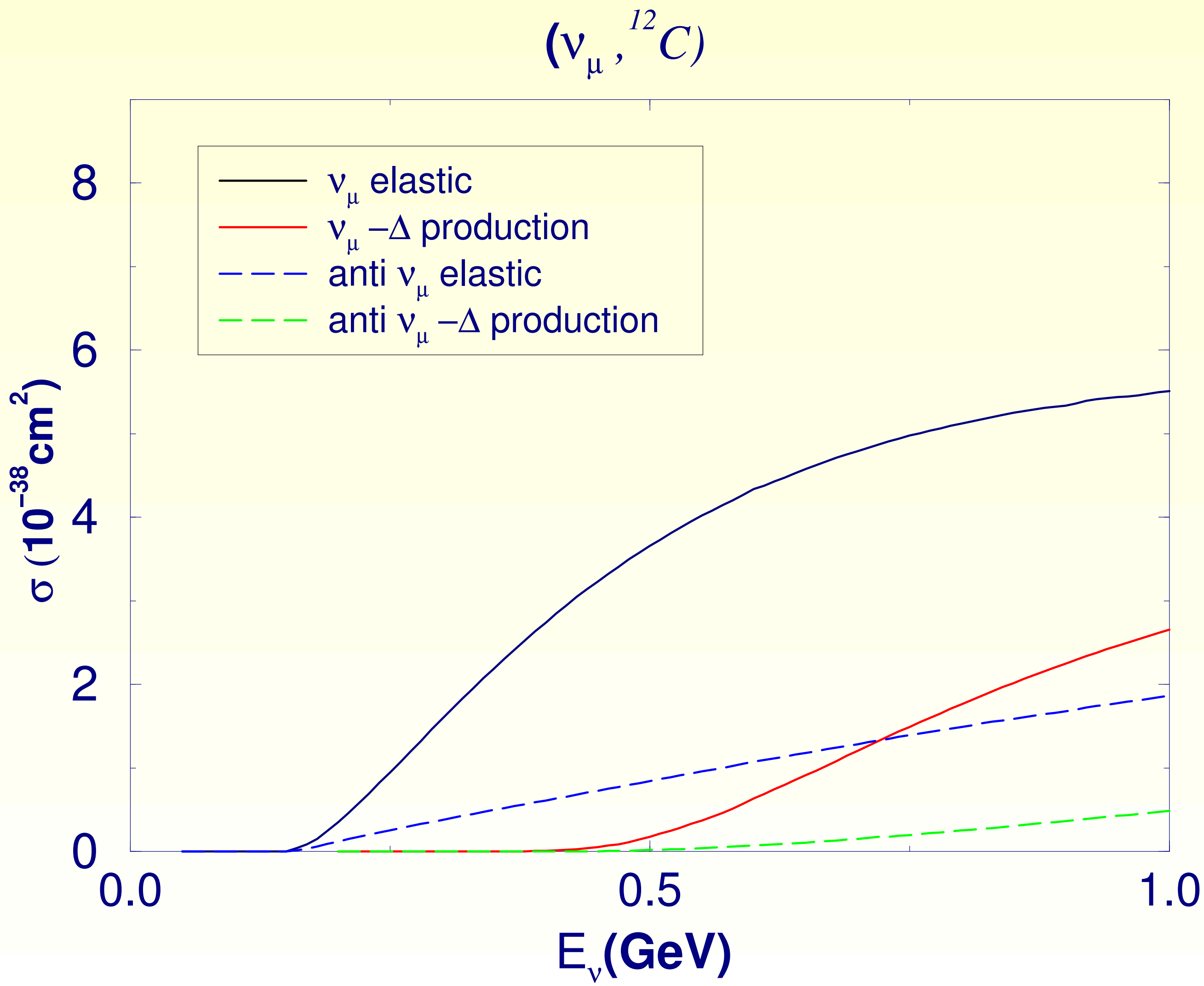
- different approach followed by C. Maieron et al., nucl-th/0303075
  Impulse approximation to describe QE scattering +
  Relativistic Shell Model to describe the bound nucleon +
  Relativistic Optical Potential (ROP) or Relativistic Mean Field (RMF) to account for FSI



good agreement with cross sections including SF + Pauli blocking

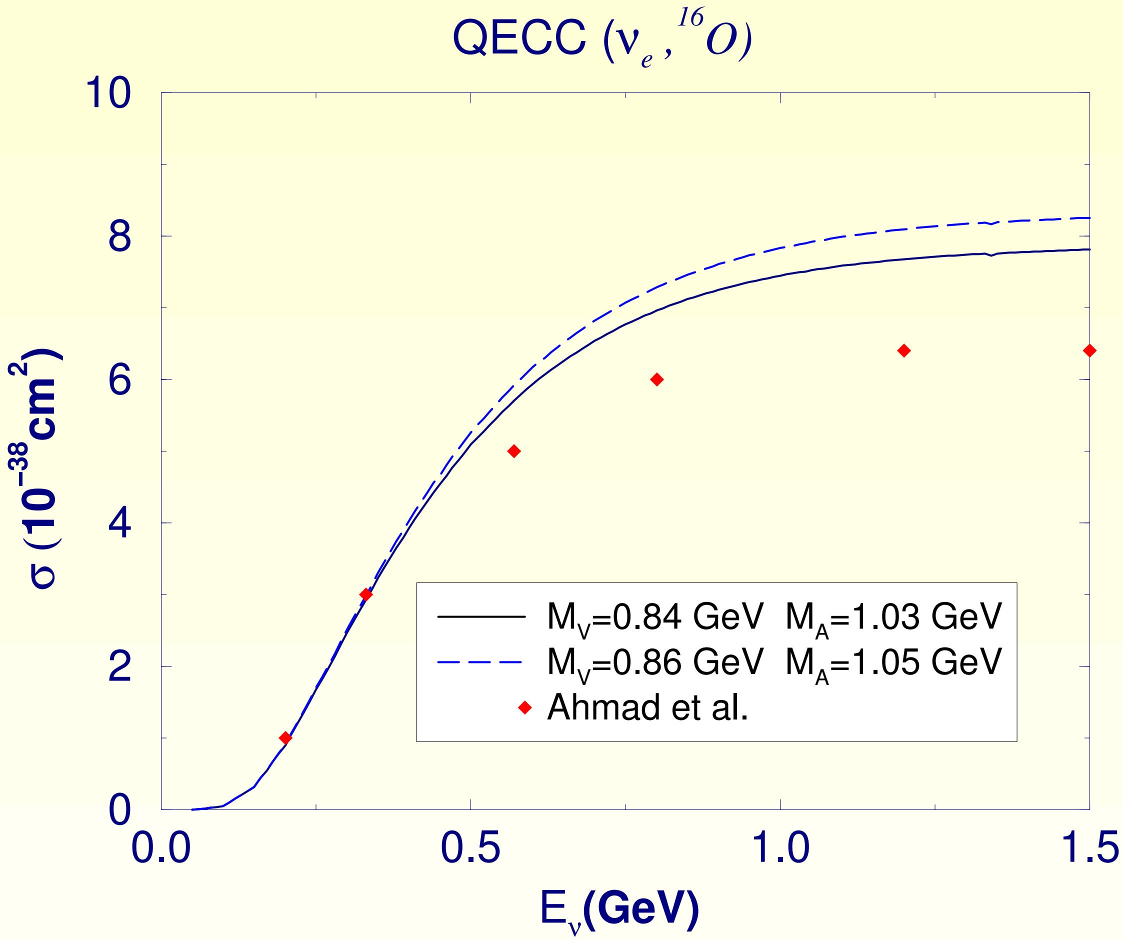
$(\nu_\mu, {}^{12}C)$

cross section of muon neutrinos on Carbon



Elastic interactions
Comparison with other calculations

- the extension at higher energies: S. Ahmad et al., nucl-th/0603001



- **we tried with two different vector and axial vector masses**
- **some discrepancy is found at energies $E_\nu > 0.5$ GeV not completely understood**

Some detail of the spectral function calculation

$$
\begin{aligned}
\langle 0|J^\mu|X\rangle &= \frac{m}{\sqrt{\mathbf{p}_{\mathcal{R}}^2+m^2}}\langle 0|\mathcal{R},\mathbf{p}_{\mathcal{R}};\mathrm{N},-\mathbf{p}_{\mathcal{R}}\rangle \\
&\times \sum_i \langle -\mathbf{p}_{\mathcal{R}}, N|j_i^\mu|x,\mathbf{p}_x\rangle \ ,
\end{aligned}
$$

$$
\begin{aligned}
W_A^{\mu\nu} &= \sum_{x,\mathcal{R}} \int d^3p_{\mathcal{R}}\ d^3p_x |\langle 0|\mathcal{R},\mathbf{p}_{\mathcal{R}};\mathrm{N},-\mathbf{p}_{\mathcal{R}}\rangle|^2\ \frac{m}{E_{\mathbf{p}_{\mathcal{R}}}} \\
&\times \sum_i \langle -\mathbf{p}_{\mathcal{R}}, \mathrm{N}|j_i^\mu|x,\mathbf{p}_x\rangle\langle \mathbf{p}_x, x|j_i^\nu|\mathrm{N},-\mathbf{p}_{\mathcal{R}}\rangle \\
&\times \delta^{(3)}(\mathbf{q}-\mathbf{p}_{\mathcal{R}}-\mathbf{p}_x)\delta(\nu+E_0-E_{\mathcal{R}}-E_x),
\end{aligned}
$$

$$
\begin{aligned}
P(\mathbf{p},E) &= \sum_{\mathcal{R}} |\langle 0|\mathcal{R},-\mathbf{p};\mathrm{N},\mathbf{p}\rangle|^2 \\
&\qquad \times\delta(E-m+E_0-E_{\mathcal{R}})\ ,
\end{aligned}
$$

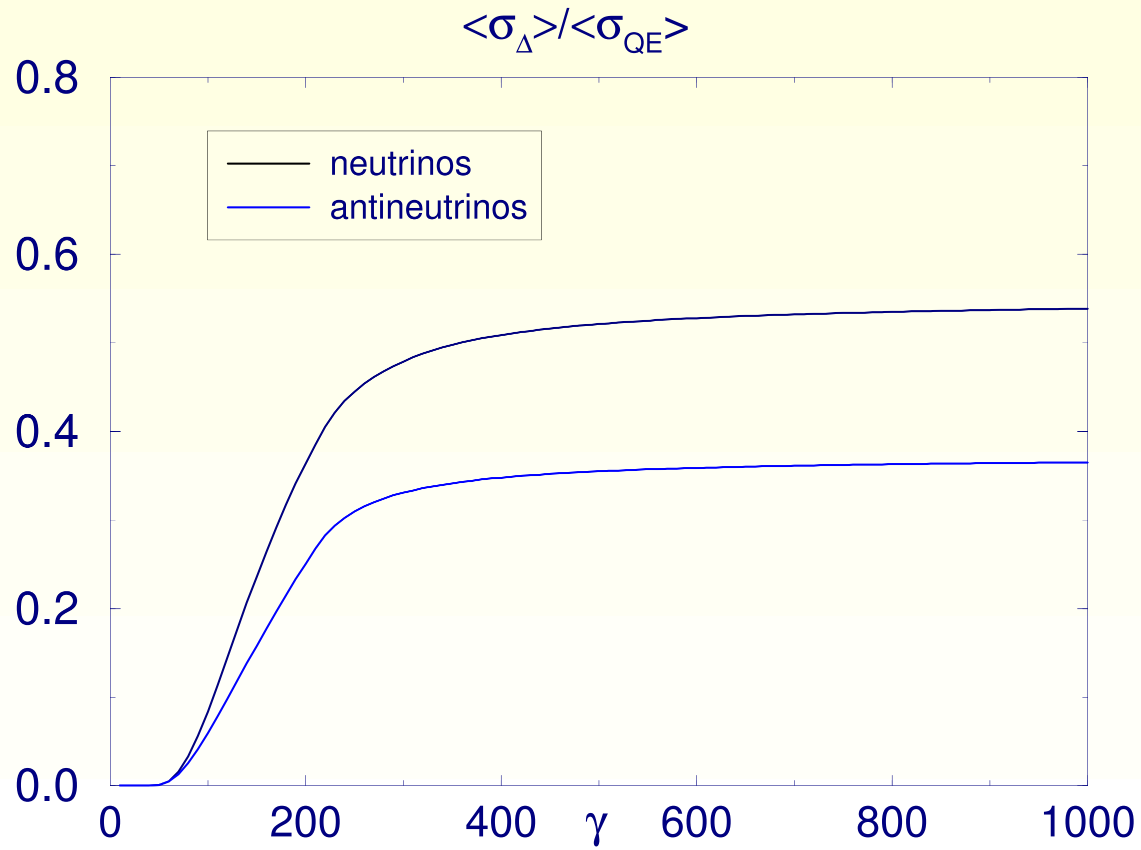
in which E_0 is the energy of the initial hadronic state. The other variables have been already introduced

Resonance production

- to evaluate the relative importance of QE and Δ-production at β-Beams, we compute (fluxes from **J. Burguet-Castell et at., Nucl.Phys.B695:217-240,2004)**

$$\bar{\sigma}_i(\gamma) \quad = \quad \frac{\int dE_\nu \, \Phi_{\nu_e}(E_\nu,\gamma)\, \sigma_i(E_\nu)}{\int dE_\nu \, \Phi_{\nu_e}(E_\nu,\gamma)} \qquad i = QE, \Delta$$

and plot the ratio $\sigma_\Delta/\sigma_{QE}$ as a function of the boost factor γ



- the contribution of Δ to the total cross section becomes constant at $\gamma > 600$
- $\bar{\sigma}_\Delta/\bar{\sigma}_{QE} \sim 0.52$ for ν
- $\bar{\sigma}_\Delta/\bar{\sigma}_{QE} \sim 0.35$ for $\bar{\nu}$